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# Particles as singularities within the unified algebraic field dynamics

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**Abstract.** We consider a nonlinear generalization of Cauchy-Riemann eqs. to the algebra of biquaternions. From here we come to "universal generating equations" (1) which deal with 2-spinor and gauge fields and form the basis of some unified algebraic field theory. For each solution of eqs.(1) the components of spinor field satisfy the eikonal and d'Alembert eqs., and the strengths of gauge field - both Maxwell and Yang-Mills eqs. We reduce eqs.(1) to that of shear-free geodesic null congruence and integrate them in twistor variables. Particles are treated as concurrent singularities of the effective metric and the electromagnetic field. For unisingular solutions the electric charge is quantized, and the metric is of Schwarzschild or Kerr type. Bisingular solutions are announced too.

## 1.Algebrodymamical approach to field theory and universal generating equations

In the general framework of *algebrodynamical* paradigm (see [1, 2, 6] and references therein) it was proposed to regard the set of equations

$$d\xi = A(x) * dX * \xi(x) \tag{1}$$

as the only basis of some unified non-Lagrangian field theory. In formula (1) the asterisk denotes multiplication in *the algebra of biquaternions* $B$ (equivalent to multiplication of matrices), and $X$ represents the $2 \times 2$ Hermitian matrix of space-time coordinates. The two-column variables $\xi(x)$ may be identified as a fundamental spinor field (related to a null complex tetrad, see Section 6), while the components $A_\mu(x)$ of the $2 \times 2$ matrix $A = A^\mu(x)\sigma_\mu$ be considered as the $C$-valued electromagnetic (EM) potentials.

The properties and interpretation of the eqs.(1) are only examined throughout this article. They originate from the *B-generalized Cauchy-Riemann equations* (Section 2), appear to be Lorentz and gauge invariant (Section 3) and impose strict limitations on both the spinor and EM fields (Section 4). Indeed, for each solution to the eqs.(1) **the components of spinor field satisfy the eikonal and d'Alembert eqs. (Sections 2,5), while the EM field strengths obey the Maxwell eqs. for free space**. Moreover, a close connection exists between the solutions to the eqs.(1) and those to the vacuum Yang-Mills and Einstein eqs. (Sections 4 and 6 respectively).

In view of the above relations between the wave-like, gauge and GTR equations (we'll call them *conventional equations* (CEqs) for brevity) the eqs.(1) have the right to be called *universal albebraic field equations* or, briefly, universal eqs. (UEqs). Since the CEqs are all of vacuum type, in the approach we develop **the particles may be regarded as nothing but** *singularities* **of the fields**. We'll see (Sections 5,7) that the structure of singularities of the CEqs (including even the free Maxwell ones) is surprisingly rich, complicated (point-, string- or toroidal-like) and possibly unknown up to now.

On the other hand, the characteristics and the time evolution of the singularities are completely governed by the *over-defined* nonlinear structure of UEqs (1), since **the CEqs are only necessary not sufficient** *compatibality conditions* **in respect to the primary system of UEqs**. In this way (Section 5), the Coulomb Ansatz corresponds to some solution of UEqs iff **the value of electric charge of the source is fixed to be unit**, this in spite of linearity of Maxwell eqs. themselves. Thus, the quantization holds here just on a classical level and due again to a strict *over-defined* structure of UEqs (1).

From the other results first presented, the relation between the UEqs and the system of *shear-free geodesic null congruence* from GTR should be distinguished. In its account, the integration of the system (1) is fulfilled in *twistor variables* (Section 6). On the other hand, this permits to define an effective *Riemann metrics* for each solution to UEqs. In the stationary axial-symmetric case, these metrics appear to satisfy the vacuum Einstein eqs. and are just of Schwarzschild or Kerr type!

In Section 7 we discuss general interpretation of particles as singularities, and bisingular solution to the UEqs is announced in this context.

**2.Algebraic origination and 2-spinor structure of universal equations**

Let $A$ be a finite-dimensional *associative* and *commutative* algebra over $R$ or $C$. The natural definition of $A$-differentiability was proposed by G.Sheffers as far as in 1893 and has the form (see [3], chapter 4 for details):

$$dF = D(Z) * dZ, \tag{2}$$

$(*)$ being multiplication in $A$, $F(Z)$ being an $A$-valued function of $A$-variable $Z \in A$, and $D(Z) \in A$ - some other $A$-function as well.

The eqs.(2) may be considered as *the condition of $A$-valued differential 1-form to be exact*[1]. For a particular case of complex algebra $A \equiv C$ the eqs.(2), after elimination of the components of $D(Z)$, lead to the Cauchy-Riemann (CR) equations of ordinary form.

To succeed in the formulation of differentiability conditions for the case of associative *noncommutative* algebra $G$ one notices that the most general component-free form of infinitesimally small increment of a $G$-function is [2]

[1]The usual conditions of smoothness of $F(Z)$ - components and existence of a positive norm in $A$-space are supposed to be fulfilled

[2]For example, in the simplest case of a quadratic function $F(Z) = Z * Z$ one has

$$dF = Z * dZ * E + E * dZ * Z,$$

$E$ being the unit element in $G$.

$$dF = L_1(Z) * dZ * R_1(Z) + L_2(Z) * dZ * R_2(Z) + ..., \tag{3}$$

the set of pairs $\{L_i(Z), R_i(Z)\}$ replacing the derivative $D(Z)$ of the commutative case. Notice that just the representation (3) serves infact as a basis of the noncommutative analysis in the version recently presented [4, chapter 7].

Unfortunately, **no relation is known to exist, generally, between the components of a "good" $G$-function**, i.e. a function whose differential may be presented in the form (3). This is quite contrary to the situation in the commutative case, in the $C$-case with CR-equations in particular. Besides, from geometrical point of view, the functions satisfying to eqs.(3) show no analogy to the conformal mappings of the complex analysis. For these reasons the version of A.Yu.Chrennikov cannot be regarded as fully successful.

The direct account of noncommutativity in the very definition of $G$-differentiability seems, however, quite natural and promising. In 1980 just this way towards the construction of noncommutative analysis was proposed by one of the authors [5] (see [1] and the references therein). On the other hand, in order to have some restrictions on the components of $F(Z)$ (*generalized CR-equations*) it was proposed to regard as "true" $G$-differentiable only such $G$-functions for which **representation (3) is reduced to one "elementary" $G$-valued differential 1-form only**, i.e. for which it holds

$$dF = L(Z) * dZ * R(Z), \tag{4}$$

$L(Z), R(Z) \in G$ being called *semi-derivatives* of $F(Z)$ (they are defined up to an element from the *centre* of $G$, see [1]).

Definition of $G$-differentiability (4) may be considered as requirement on an elementary $G$-valued 1-form to be exact [3]. For $G$ being commutative again, the conditions (4) are evidently reduced to the old ones (2) (and so to the CR-equations in $C$-case).

The definition (4) appreciably narrows down the class of "good" $G$-functions, cutting off, say, all the polynomials (exept the trivial linear one). This certainly seems rather unexpected from the standpoint of customary complex analysis. Nevertheless, it singles out just such a class of $G$-functions which is natural by algebraic considerations, extremely interesting in geometrical properties and admits a wonderful field-theoretical interpretation.

In the exclusive case of real Hamilton quaternions $G \equiv H$ eqs.(4) appear to be just **the algebraic condition for the mapping $F : Z \to F(Z)$ to be *conformal* in $E^4$** (see [1], [6] for details). However, since the conformal group of $E^4$ is known to be finite (15-) parametric, the functions corresponding to eqs.(4) are rather trivial to be treated, say, as field variables. Fortunately, the situation becomes quite different when one turns **to the complex extention of $H$, i.e. to the algebra of *biquaternions* $B$**, which only we are going to deal with below.[4]

For $B$-algebra the $2 \times 2$ complex matrix representation is suitable. For it we take

$$Z \Leftrightarrow \begin{pmatrix} z^0 + z^3 & z^1 - iz^2 \\ z^1 + iz^2 & z^0 - z^3 \end{pmatrix} \equiv \begin{pmatrix} u & \bar{w} \\ w & v \end{pmatrix} \equiv z^\mu \sigma_\mu, \tag{5}$$

[3]Note that the *elementary* $G$-form may be defined as *the most general $G$-valued 1-form* which may be constructed by means of operation of multiplication in $G$ only

[4]Some considerations about differentiability in Dirac-Clifford and octonion algebras are presented in [1, chapter 1]

where $z^\mu \in C$ and $\sigma_\mu = \{E,\ \sigma_a\}$ being the unit and three Pauli matrices respectively (as usual, $\mu, \nu, ... = 0, 1, 2, 3$ and $a, b, ... = 1, 2, 3$), and $u, v = z^0 \pm z^3;\ w, \bar{w} = z^1 \pm iz^2$ being spinor coordinates to be used below). Applying now the column or the full row-column splitting to the eqs.(4) we come to the following two forms:

$$d\xi = L(Z) * dZ * \eta(Z), \qquad df = \phi(Z) * dZ * \psi(Z), \tag{6a, b}$$

$\xi(Z), \eta(Z) \in C^2$ being two 2-columns and $\phi(Z), \psi(Z) \in C^2$ - two 2-rows, mutually independent in general, $f(Z) \in C$ - some any matrix component of $F(Z)$. According to the symmetry properties of the eqs.(6a,b) the quantities $\xi, \eta, \phi, \psi$ manifest themselves as 2-spinors, whereas $f(Z)$ as a scalar (see Section 3 and the article [7] for details).

From the condition (6b) in account of well-known *Fiertz identities*, **the (complexified) eikonal equation for each (matrix) component $f(Z)$ of a $B$-differentiable function $F(Z)$ now follows** [8, 1]

$$\eta^{\nu\lambda} \partial_\nu f \partial_\lambda f = 0, \tag{7}$$

$\partial_\nu \equiv \partial / \partial z^\nu$ being the partial derivatives and $\eta^{\nu\lambda}$ being the metric tensor, of the form $\eta^{\nu\lambda} = diag(+1, -1, -1, -1)$. in representation (5).

The eikonal equation (7) plays in $B$-analysis the role similar to that of the Laplace equation in two-dimensional complex case. Thus, the definition (4) of $G$-differentiability links together the *noncommutativity* of $G$-algebra (figurating directly in (4)) with the *nonlinearity* of the generalized CR-equations consequent. There is nothing surprising in this correlation from the usual standpoint of gauge theories (non-Abelian group results in a nonlinear structure of Yang-Mills strengths). However, within the framework of the noncommutative analisis this interrelation was proposed and demonstrated, perhaps, for the first time (all of the previous works deal with trivial linear generalizations of the CR-equations, see for example [3, 9]).

Noticing that the eqs.(6b) follow directly from the eqs.(6), and the latters make it possible to reconstruct an arbitrary solution to the entire system (4), we come to the *fundamental 2-spinor structure* for the primary system (4) to possess. Together with its nonlinear character this allows us to formulate a field theory on the base of eqs.(6a) only. We hope to do this elsewhere, whereas here we restrict ourselves with a particular case $\xi(Z) \equiv \eta(Z)$ in eqs.(6a).

Aa the only *ad hoc* conjecture we are obliged to take here is the requirement for coordinates $z^\mu$ in (5) to be real, $z^\mu \equiv x^\mu \in R$, i.e. to belong to the *Minkowsky space* being a subspace of the entire complex vector space of $B$-algebra (this requires for the coordinate-representing matrices in (5) to be Hermitian, $Z \equiv X = X^+$).

In account of these two limitations, the second one clearly necessary to ensure the *relativistic invariance* of theory, we come back to the UEqs (1), announced at the beginning, as the equations of some **algebraic nonlinear field theory**, having something to do with a spinor field as well as with gauge fields (see next Section). It is obvious, however, that such a theory will be very exotic due to the **over-defined, nonLagrangian structure** of its only dynamical background - the UEqs, to which we now proceed.

## 3.Geometrodynamical interpretation and gauge structure of UEqs.

In a 4-index form the UEqs (1) take the form

$$\partial_\nu \xi = A(x) * \sigma_\nu * \xi(x), \tag{8}$$

where $A(x) \equiv L(X) = A^\mu(x)\sigma_\mu$. According to (1) or (8), the UEqs. may be considered in the framework of geometrodynamics as **the conditions for a fundamental spinor field** $\xi(x)$ **to be covariantly constant** in respect to the effective affine connection

$$\Gamma = A(X) * dX, \qquad \Gamma_\nu = A(x) * \sigma_\nu \tag{9}$$

which may be called *left B-connection*. It is determined by the structure of $B$-algebra and induces a specific affine geometry of *Weyl-Cartan type* on the complex vector space of $B$-algebra. To see this, one should return back in (8) from the spinor $\xi(X)$ to a full $2 \times 2$ matrix $F(X) = F^\mu(x)\sigma_\mu$; then one gets for the components

$$\partial_\nu F^\mu = \Gamma^\mu_{\nu\rho}(A(x))F^\rho(x), \tag{10}$$

where the connection coefficients have a specific form

$$\Gamma^\mu_{\nu\rho}(A) = \delta^\mu_\nu A_\rho + \delta^\mu_\rho A_\nu - \eta_{\nu\rho}A^\mu - i\varepsilon^\mu_{.\nu\rho\lambda}A^\lambda, \tag{11}$$

including the *Weyl nonmetricity* and the *torsion* terms related to each other (the Weyl's vector $A_\mu(x)$ being proportional to the pseudo-trace of the torsion tensor $iA_\mu(x)$).

Note that the $B$-induced complex Weyl-Cartan connection (11) was proposed firstly in [1] and recently used by V.G.Kretchet in his search for geometric theory of electroweak interactions [10] (based on the break of $P$-invariance by the torsion term in (11)).

The UEqs (1) are evidently form-invariant under the global transformations of coordinates and field variables

$$X \Rightarrow X' = M^+ * X * M, \qquad \xi \Rightarrow \xi' = M^{-1}\xi, \quad A \Rightarrow A' = M^{-1} * A * (M^+)^{-1}, \tag{12$a,b$}$$

$M \in SL(2, C)$ being an arbitrary unimodular ($\det \|M\| = 1$) $2 \times 2$ complex matrix.

The 6-parametric group of transformations (12a) $2:1$ corresponds to the continious transformations of the coordinates $x^\mu$ ($X = x^\mu\sigma_\mu$) from Lorentz group. Thus, the **UEqs. are relativistic invariant** and, according to the laws of transformations (12b), the quantities $A_\mu(x)$ and $\xi_B(x), B = 1, 2$ behave themselves as the components of a 4-vector and a 2-spinor respectively. As to the local symmetries of UEqs., the system (8) may be showm to preserve its form under so called "*weak gauge transformations*" [7]

$$\xi_B \Rightarrow \xi'_B = \lambda\xi_B, \qquad A_\mu \Rightarrow A'_\mu = A_\mu + \frac{1}{2}\partial_\mu \ln\lambda, \tag{13}$$

$\lambda \equiv \lambda(\xi^1, \xi^2) \in C$ being some rather smooth scalar function **dependent on two spinor components of the starting solution only** (instead of its direct dependence on all 4-coordinates $x^\mu$ in a usual gauge approach)![5]

In account of the gauge nature of the 4-vector $A_\mu(x)$ and of its close relation to the Weyl's nonmetricity vector, it seems quite natural **to identify** $A_\mu(x)$ (up to a dimensional factor) **with the 4-vector of potentials of a ($C$-valued) electromagnetic field**. Leaving for

[5] Besides, the UEqs. are invariant under the gauge transformations of Weyl type, related to the conformal change of metric; a discussion of the *double gauge group* so arising may be found in [2].

the next Section the discussion of complex structure of EM-field, we recall only that both the spinor and EM fields may be obtained from the only system (8) due to its over-defined structure. Then the problem arises what sort of restrictions on EM-strengths are imposed by UEqs, and in what way are they related to Maxwell equations?

**4.Self-duality, Maxwell & Yang-Mills equations as the consequences of UEqs.**

Since the set of UEqs (1) or (8) is over-defined (8 eqs for 2 spinor plus 4 potential components), some *compatibility conditions* should be satisfied. In particular, commutators of partial derivatives $\partial_{[\mu}\partial_{\nu]}\xi = 0$ in (8) should turn to zero, this being correspondent to the *closeness* of the $B$-valued 1-form in (1) owing to the Poincaré lemma. After the calculations being derived we get

$$0 = R_{[\mu\nu]}\xi, \tag{14}$$

where the quantities

$$R_{[\mu\nu]} = \partial_{[\mu}A\sigma_{\nu]} - [A\sigma_\mu,\ A\sigma_\nu] \tag{15}$$

represent the *$B$-curvature tensor* of the $B$-connection (9).

From (14) it doesn't follow $R_{[\mu\nu]} \equiv 0$, since the spinor $\xi(x)$ is not an arbitrary one. However, it may be shown (see [1], [7]) that the *self-dual part* $R^+_{[\mu\nu]}$ of (15)

$$R^+_{[\mu\nu]} \equiv R_{[\mu\nu]} + \frac{i}{2}\varepsilon_{\mu\nu}^{\cdot\cdot\rho\lambda}R_{[\rho\lambda]} = 0 \tag{16}$$

should turn to zero as a consequence of eqs.(14). Being written in components, the expressions (15),(16) result in 3+1 equations

$$\mathcal{F}_{[\mu\nu]} + \frac{i}{2}\varepsilon_{\mu\nu}^{\cdot\cdot\rho\lambda}\mathcal{F}_{[\rho\lambda]} = 0, \tag{17}$$

$$\partial_\mu A^\mu + 2A_\mu A^\mu = 0, \tag{18}$$

where the tensor

$$\mathcal{F}_{[\mu\nu]} = \partial_{[\mu}A_{\nu]} \tag{19}$$

is a usual strengths' tensor of EM field. 3-vector form of the eqs.(17)

$$\vec{\mathcal{E}} + i\vec{\mathcal{H}} = 0 \tag{20}$$

relates the ($C$-valued) *electric* $\vec{\mathcal{E}}$ and *magnetic* $\vec{\mathcal{H}}$ vectors of field strength tensor

$$\mathcal{E}_a = \mathcal{F}_{[oa]} = \partial_o A_a - \partial_a A_o, \quad \mathcal{H}_a = \frac{1}{2}\varepsilon_{abc}\mathcal{F}_{[bc]} = \varepsilon_{abc}\partial_b A_c. \tag{21}$$

Thus, we see that **the self-duality conditions (17) and the "inhomogeneous Lorentz condition" (18)** [6] **appear to be the integrability conditions of the UEqs.**

According to the definitions of field strengths through the potentials (21) and to the self-duality conditions (20), we conclude then that **the free-space Maxwell equations are satisfied identically for each solution to the UEqs!**

[6] Geometrically it corresponds to the condition for the scalar 4-curvature invariant of the Weyl tensor to be null, see [7]

The complex nature of field strengths (21), however, doesn't manifest itself in doubling of the degrees of freedom' number of EM field owing just to the self-duality eqs.(20); from the latters we get only

$$\vec{B} = \vec{E}, \qquad \vec{D} = -\vec{H}, \tag{22}$$

$\{\vec{E}, \vec{H}\}$ and $\{\vec{D}, \vec{B}\}$ being respectively the real ($\Re$) and imaginary ($\Im$) parts of the primary complex fields $\{\vec{\mathcal{E}}, \vec{\mathcal{H}}\}$. The real-part fields $\vec{E}$ and $\vec{H}$ are therewith mutually independent in algebraic sense and satisfy the Maxwell eqs. owing to linearity of the latters. [7].

The meaning of decomposition of the unique complex field into the real and imaginary parts is that the density of conservative *energy-momentum tensor* can be defined through the latters in a usual way, while for the complex fields the related quantities

$$w \propto \vec{\mathcal{E}}^2 + \vec{\mathcal{H}}^2, \qquad \vec{p} \propto \vec{\mathcal{E}} \times \vec{\mathcal{H}} \tag{23}$$

vanish in account of the self-duality conditions (20). Some preferance of the $\Re$-part fields $\vec{E}, \vec{H}$ may be therewith justified from geometrical considerations (see Section 5).

In addition to all this it may be shown [7] that the structure of the UEqs, and of the $B$-connection (9) in particular, make it possible to define a $C$-valued *Yang-Mills field.* Infact, for the connection (9) one obtains the expression

$$\Gamma_\nu = A(X) * \sigma_\nu = A^\mu(x)\sigma_\mu * \sigma_\nu = A^\mu(x) B^\rho_{\mu\nu}\sigma_\rho \equiv A_\nu(x) + N^a_\nu(x)\sigma_a, \tag{24}$$

$B^\rho_{\mu\nu}$ being *the structure constants* of $B$- algebra. Then the *trace-free-part* variables $N^a_\nu(x)$,

$$N^a_o = A_a(x); \qquad N^a_b = \delta_{ab}A_o(x) + i\varepsilon_{abc}A_c(x) \tag{25}$$

may be identified with the potentials of some Yang-Mills (YM) field of a special type.[8]

The trace-free part of $B$-curvature tensor (15) gives then for the YM field strengths

$$\mathcal{L}^a_{[\mu\nu]} = \partial_{[\mu}N^a_{\nu]} - \frac{i}{2}\varepsilon_{abc}N^b_\mu\, N^c_\nu. \tag{26}$$

For a nonzero solution $\xi(x)$ it follows now from eqs.(14) *for each of* $[\mu\nu]$ *component*

$$\det \|R_{[\mu\nu]}\| \equiv \mathcal{F}^2_{[\mu\nu]} - \mathcal{L}^a_{[\mu\nu]}\mathcal{L}^a_{[\mu\nu]} = 0, \tag{27}$$

In view of (27) **EM field (21) should be regarded as a modulus of isotopic vector of YM-triplet field**, both fields being described through a unique $B$-connection (9) and the EM field being correspondent to the trace part of it.

Such an interrelation between EM and YM fields proposed firstly in [7] is gauge invariant and requires no participation of a chiral field as in usual gauge approach. However, the subset of YM fields (25) can't be *pure*, being always accompanied by an EM field, due to the definite norm of the isotopic 3-space (see (27)).

[7]The same being true, of course, for the $\Im$-part fields $\vec{D}, \vec{B}$, providing in account of (22) a dually-conjugate solution to the Maxwell eqs.

[8]The YM potentials like (25) belong to the class of so called *invariant* fields

All of the above speculations would be significant if only the YM eqs. would be satisfied for the field variables (25),(26). Fortunately, it is just the case, since the trace-free part of the self dual $B$-curvature (15) includes only the corresponding self-dual configuration of Maxwell strength tensor and the Lorentz inhomogeneous form, both being null in account of the integrability conditions (17),(18). Thus, for each solution to the UEqs (8) **the YM field strengths (26) are self-dual and satisfy therefore the YM eqs. for free space**.

It may be noted in conclusion that, contrary to the EM case, the $\Re$ and $\Im$ parts of the $C$-valued strengths (26) won't satisfy YM eqs. separately; so the YM fields considered are essensially complex in nature! On the other hand, it may be checked that the YM strengths (2) preserve non-Abelian (commutator) part for the potentials of the form (25). [9]

**5.Dion-like unisingular solution and quantization of electric charge**

The vacuum Maxwell eqs. hold identically for each solution to the generating UEqs. Hence, no soliton-like field distributions can exist for the model considered. Nevertheless, **the particles may be brought into correspondence with the singular points (or strings, membranes etc.) of the field functions**, in which the $B$-differentiability conditions are violated.

In account of the self-dual structure of gauge fields proved, **charged singular solutions**, if exists, **should be dions**, i.e. carry both the electric and magnetic charges of equal (up to a factor "i") magnitudes! Elementary unisingular dion-like solution has been found in [1]. To obtain it here, we can fix the gauge so that to have for the 2-spinor $\xi(x)$ the form

$$\xi^T(x) = (\ 1,\ G(x)\ ); \tag{28}$$

then for the EM potentials one gets

$$A_w = \partial_u G, \quad A_v = \partial_{\bar{w}} G, \quad A_u = A_{\bar{w}} \equiv 0, \tag{29}$$

and the system of UEqs (8) is reduced to a couple of nonlinear differential eqs. for a unique unknown function $G(x)$ only

$$\partial_w G = G\partial_u G, \qquad \partial_v G = G\partial_{\bar{w}} G, \tag{30}$$

where the *light-cone coordinates* $\{u, v, w, \bar{w}\}$ defined previously by eq.(5) have been used.

By mutual multiplication of eqs.(31) we come then to the connection

$$(\partial_u G)(\partial_v G) - (\partial_w G)(\partial_{\bar{w}} G) = 0, \tag{31}$$

which appears to be just the eikonal eq.(7) in spinor coordinates. Now we obtain the commutator of derivatives in the left side of eqs.(30) and get, in respect to eq.(31),

$$\partial_u \partial_v G - \partial_w \partial_{\bar{w}} G = 0, \tag{32}$$

the latter being just the wave d'Alembert equation $\bigtriangledown^2 G = 0$. It may be shown that this result is gauge invariant, so that **each component of the 2-spinor field $\{\xi_B(x)\}$ obeys both the eikonal and d'Alembert eqs. (31),(32)!**

[9] As to the possible nonAbelian nature of EM field itself, it was discussed recently in [11], also in the framework of Weyl geometry

Fundamental non-trivial solution common to eqs.(31),(32) found in [1], corresponds to the *stereographic mapping* $S^2 \mapsto C$ of the 2-sphere onto the complex plane:

$$G = \frac{x^1 + ix^2}{x^3 \pm r} \equiv \frac{\bar{w}}{z \pm r} \equiv \tan^{\pm 1}\frac{\theta}{2}\exp^{i\varphi}, \tag{33}$$

$\{r, \theta, \varphi\}$ being usual spherical coordinates. From the solution (33), which satisfy the couple of eqs.(30) under consideration, using the expressions (29) the EM potentials $A_w, A_v$ may be found; then for the scalar ($A_o$) and spherical components we get

$$A_o = \pm\frac{1}{4r}, \quad A_r = -\frac{1}{4r}, \quad A_\varphi = \pm i A_\theta = \frac{i}{4r}\tan^{\pm 1}\frac{\theta}{2} \tag{34}$$

Now for the nonzero components of $C$-valued EM field strengths (21) we get (the electric field appears to be pure real, while the magnetic - pure imaginary!)

$$\mathcal{E}_r = \pm\frac{1}{4r^2}, \quad \mathcal{H}_r = \pm\frac{i}{4r^2}, \tag{35}$$

(note that the components $A_r,\ A_\theta$ don't contribute into the magnitude of field strengths, being of a pure gauge type). Hence, the fundamental unisingular solution (33) corresponds to **a point source with a fixed value of electric charge** $q = \pm 1/4$ **and an equal (imaginary) value of magnetic charge** $m = \pm i/4$.

At this stage of consideration, the factor (1/4) isn't of great importance, since the *physical* EM potentials were determined up to an arbitrary dimensional factor only. What is really significant is that 1) all values of electric charge except *the only possible one* are not allowed for the point particle-like source to possess, and 2) its Coulomb field is always accompanied by the magnetic monopole field with the charge equal to the electric one!

We set aside the general problem of charge quantization in this and similar (see [2]) models in hope to discuss it elsewhere, and consider now an interesting modification of the solution (33)-(35), which can be obtained through the *complex translation* $z \mapsto z + ia,\ \ a \in R$, the latter being obviously a symmetry of UEqs. By this we come to a new solution, whose electric field structure instead of Coulomb form (35) corresponds to a known Appel solution (see e.g. [12]). The singular EM -field structure will be then defined by the condition

$$r^* \equiv \sqrt{(z + ia)^2 + x^2 + y^2} = 0, \quad \Rightarrow \quad \{x^2 + y^2 = a^2, \quad z = 0\}, \tag{36}$$

corresponding to a *ring-like* source of radius $|a|$. For the real-part fields ($\Re$-fields) $\{\vec{E}, \vec{H}\}$ the following asipmtotic expressions at the distances $r >> |a|$ are true:

$$E_r \simeq \frac{q}{r^2}(1 - \frac{3a^2}{2r^2}(3\cos^2\theta - 1)),\ E_\theta \simeq -\frac{qa^2}{r^4}3\cos\theta\sin\theta,\ H_r \simeq \frac{2qa}{r^3}\cos\theta,\ H_\theta \simeq \frac{qa}{r^3}\sin\theta. \tag{37}$$

In view of eqs.(36)-(37), the $\Re$-field solution is related to the singular ring with a quantized value of electric charge $q = \pm 1/4$, *dipole magnetic moment* $\mu = qa$ and *quadrupole electric moment* $\vartheta = -2qa^2$. If we assume now for $|a|$ the value $|a| = \hbar/2Mc$ in order to have for the magnetic moment the known Dirac value $\mu = e\hbar/2Mc$, then it may be conjectured

**for a fundamental charged fermion to possess necessarily a quadrupole electric momemt** $\vartheta$ equal in magnitude to

$$\vartheta = \frac{e\hbar^2}{2M^2c^2} \tag{38}$$

At present, such a statement looks rather speculative; nontheless, the possibility of its experimental prove may be discussed. However, much more fundamental seems to be the fact that for the $\Re$-part fields their asimptotic structure (37) is in complete agreement with that observed for elementary particles, whereas the $\Im$-fields contains only the "phantom" terms proportional, say, to the magnetic charge or to the dipole electric moment!

Geometrically, the phantom-like $\Im$-fields contribute only into the *torsion* terms of the "Minkowsky-projection" of the complex $B$-connection (9). Then, owing to a specific (totally skew symmetric *Rodichev-type*) structure of the torsion considered, the $\Im$-fields won't enter into the eqs. of *geodesics*, this resulting in **total non-observability of magnetic charges and electric dipole moments for the elementary particles** (see [7] for details).

**6.Effective metric, twistor variables and implicit general solution to UEqs**

In a special gauge the UEqs were shown to reduce themselves to the couple of eqs.(30). The latter is known in GTR as the system defining a *shear-free geodesic null congruence* (SFGNC) $l_\mu(x)$, for which we can take, say, the spinor representation

$$l_\mu \Leftrightarrow L = \begin{pmatrix} 1 & \bar{G} \\ G & \bar{G}G \end{pmatrix}, \quad \det \|L\| \equiv 0. \tag{39}$$

Then the induced Riemannian metric of a *Kerr- Schild type*

$$ds^2 = dudv - dwd\bar{w} - M\Re(\partial_{\bar{w}}G)(du + Gd\bar{w} + \bar{G}dw + G\bar{G}dv)^2/(1+G\bar{G}) \tag{40}$$

in stationary case satisfy the vacuum Einstein equations [13, 15]. For unisingular solution above-presented the metric (40) **appear to be just of Schwarzschild (for the point singularity (33)) or Kerr (for the ring singularity (36)) types!**

We pass now to the demonstration of complete integrability of the system (30) based on a famous Kerr theorem (see [14, chapter 7]). In respect to it, the general solution to the SFGNC system of eqs.(30) has the form of implicit dependence of $G(x)$

$$F(G, \tau_1, \tau_2) = 0 \tag{41}$$

upon the (projective) *twistor variables*

$$\tau = X\eta \qquad (\tau_1 = u + wG, \quad \tau_2 = \bar{w} + vG). \tag{42}$$

The *caustic* condition

$$\frac{dF}{dG} = 0 \tag{43}$$

is ten known to define the singular points (or strings etc.) of the curvature of the Kerr-Schild metric (40). On the other hand, **the same condition (43) may be verified to fix the singularities of EM-field** which can be constructed from the $G(x)$-function through the 4-potentials (29).

Thus, we reduced the UEqs (8) to the SFGNC-system (30) and the latter - to a purely algebraic problem of resolving of the implicit-functional dependence (41). The last problem, rather complicated in its turn, will be discussed elsewhere.

**6.Perestroikas of singularities as mutual transmutations of particles**

The structure of UEqs (1), purely algebraic in origin and compact in form, appears to be very complicated, being related to spinors, twistors and gauge fields. Each solution to (1) satisfy, in particular, Maxwell eqs., whose singular structure is found to be quite nontrivial (see below) and may be therefore identified with that of elementary particles.

On the other hand, not an every solution to the linear Maxwell eqs. corresponds to some any solution to the primary system (1), from where the quantization of electric charge does follow, as well as the nontrivial time evolution of the particles-singularities.

From a purely mathematical point of view, all this bears a direct relationship with the rapidly progressing *catastrophe theory* [16], in whose framework **the "perestroikas" of singularities should be interpreted as the processes of mutual transmutation of elementary particles**.

The confirmation for such a conjecture we have ontained recently [17], where an exact bisingular solution to the UEqs (and, hence, to the ordinary Maxwell eqs. as well) was presented. Its structure describes the axial-symmetric interaction of two point-like oppositely charged singularities, the magnitudes of charges being equal to the charge of unisingular solution (35). Under some values of integration constant, this solution describes also the *"creation - annihilation" processes* of particles - singularities, together with *an intermediate resonance structure of a toroidal geometry*. We hope to examine it in detail in the near future, as well as the general problem of interactions of singularities in the framework of the UEqs model, which may give rise to many other striking phenomena.

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